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Schematising in Early Childhood Mathematics Education: Why, When and How?

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SUMMARY: Discussions about the beginnings of mathematics education in early childhood are often caught up in a dilemma: should we stimulate spontaneous actions or provide direct instruction about elementary number-related actions? In this article we argue that either approach is problematic and neither is an optimal way of promoting the development of mathematical thinking. Using a socio-cultural perspective, we propose an emergent approach that integrates both the child's personal constructions and the educator's pedagogical responsibilities. From this stance, we conceptualise mathematical thinking as a form of semiotic activity. Early semiotic activity can be identified in schematising activities in early childhood play. Educationally important questions are then: What are our arguments for introducing schematising activities in early childhood education? And how can educators organise schematising activities in early childhood education?

This article gives an overview of some theoretical and empirical arguments drawn from learning theory and developmental theory. It also provides theory-driven descriptions of exemplary practices that can be seen as promising starting points for early mathematics education.

RÉSUMÉ: Les discussions sur les commencements de l'enseignement mathématique à l'école maternelle débouchent souvent sur un dilemme : faut-il stimuler les actions spontanées des jeunes enfants ou faut-il instruire explicitement les activités numériques? Dans cet article, nous montrons que ni l'une ni l'autre approche n'est une démarche optimale pour stimuler le développement de la pensée mathématique. Partant d'un point de vue socioculturel, nous proposons une approche naissante dans laquelle les constructions personnelles de l'enfant et les responsabilités pédagogiques de l'enseignant ont été intégrées. Dans ce cadre, nous considérons la pensée mathématique comme une forme d'activité sémiotique. Les premières activités sémiotiques peuvent être observées dans les activités de schématisation dans le jeu des jeunes enfants. Les questions pédagogiques importantes sont alors : Quelles sont nos arguments en faveur de l'introduction des activités de schématisation dans l'éducation à l'école maternelle? Et comment les éducateurs peuvent-ils organiser ces activités dans la pratique scolaire quotidienne?

Cet article donne une vue d'ensemble de quelques arguments théoriques et empiriques empruntés à la théorie de l'apprentissage et à celle du développement. De plus, il présente quelques descriptions d'exemples pratiques qui peuvent être vus comme des points de départ précieux à l'éducation mathématique à l'école maternelle.

ZUSAMMENFASSUNG: Diskussionen über den Beginn mathematischer Bildung in der frühen Kindheit stecken oft in einem Dilemma: Sollen spontane Aktionen angeregt oder direkte Instruktionen zu elementaren zahlenbezogenen Aktionen gegeben werden? Es wird ausgeführt, dass jeder dieser beiden Ansätze problematisch ist und keiner den optimalen Weg zur optimalen Förderung der Entwicklung mathematischen Denkens bietet. Unter Heranziehung einer

soziokulturellen Perspektive wird ein neuer Ansatz vorgeschlagen, der sowohl die persönlichen Konstruktionen des Kindes als auch die pädagogischen Verantwortlichkeiten des Pädagogen integriert. Mathematisches Denken wird hier als eine Form semiotischer Aktivität konzeptualisiert. Frühe semiotische Aktivität kann in schematisierenden Handlungen in frühem kindlichem Spiel identifiziert werden. Daraus folgende wichtige Fragen für die Bildung lauten dann: Welche Gründe gibt es dafür, schematisierende Aktivitäten in die Bildung der frühen Kindheit einzuführen? Und wie können Pädagogen diese organisieren?

Der Artikel gibt einen Überblick über einige theoretische und empirische Argumente aus der Lerntheorie und der Entwicklungstheorie. Theoriegeleitete exemplarische Praxis wird beschrieben, die als viel versprechende Startpunkte für frühe mathematische Bildung angesehen werden können.

RESUMEN: *Las discusiones sobre el comienzo de la educación en matemáticas en la infancia temprana se ven a menudo atascadas en un dilema: ¿Deberíamos estimular acciones espontáneas o dar instrucciones directas sobre las acciones elementales relacionadas con números? En este artículo defendemos que ambas aproximaciones son problemáticas y que ninguna es un camino óptimo para promover el desarrollo del pensamiento matemático. Usando una perspectiva socio-cultural, proponemos una aproximación emergente que integra tanto las construcciones personales del niño como las responsabilidades pedagógicas del educador. Desde esta posición, conceptualizamos el pensamiento matemático como una forma de actividad semiótica. La actividad semiótica temprana puede ser identificada en actividades esquematizantes en el juego en la infancia temprana. Entonces, cuestiones educacionalmente importantes son: ¿Cuáles son nuestros argumentos para introducir actividades esquematizantes en la educación en la infancia temprana? Y ¿Cómo pueden los educadores organizar actividades esquematizantes para la educación en la infancia temprana?*

Este artículo da una visión general de algunos argumentos teóricos y empíricos traídos de la teoría del aprendizaje y de la teoría del desarrollo. También aporta descripciones guiadas por la teoría de prácticas ejemplares que pueden ser vistas como prometedores puntos de partida para la educación matemática temprana.

Keywords: Mathematics; Schematising; Early childhood; Play; Socio-cultural.

Traditional versus progressive

"The history of educational theory is marked by opposition between the idea that education is development from within and that it is formation from without; that it is based upon natural endowments and that education is a process of overcoming natural inclination and substituting in its place habits acquired under external pressure." (Dewey, 1997b)

The polarisation between *traditional* and *progressive* or *child centred* and *subject centred* education has sharply marked the discussions about the organisation of educational theory and practice. Dewey (1859-1952) was one of the most influential thinkers who argued that both approaches have valuable elements and neither is sufficient by itself, but must somehow be fused (Dewey, 1997a; 1997b; see also Egan, 1988). Although the original discussions go beyond the currently well-known domains of the curriculum, we perceive the same debates today within each subject matter taught in schools and maybe even for each age-group. Take, for example, the discussions about the beginnings of mathematics education in early childhood education. These discussions are often caught up in the dilemma of stimulating spontaneous actions of children, on the one hand, and directly instructing elementary number-related actions of children, on the other. Following Dewey, many researchers and practitioners in the field of mathematics education in early childhood have tried to find ways to integrate both views in one coherent educational system, by taking a third trail in order to promote the development of mathematical thinking in young children.

In this article, we defend an approach that has emerged in Dutch education in the last decades and that follows such a third trail. This approach is called Developmental Education and

relies heavily upon the ideas of Vygotsky and his colleagues. Children learn elementary related number actions within meaningful contexts. They learn the formal symbolisations of mathematics through their own inventions of meaningful symbols and notations, through interaction, discourse, schematising and play under adult guidance. In this view, mathematical thinking is conceptualised as a form of thinking about quantitative and spatial relationships with the help of symbolic means. The construction of symbolic forms (like schemes, diagrams, drawings) and the reflection on the interrelationships between these forms and their meanings is essential for mathematical thinking. For that reason, we conceptualise mathematising as a form of semiotic activity.

Here, we will focus on early semiotic activity of 5-7 year-olds that can be identified in schematising activities in their play. We believe that schematising activities in early childhood can stimulate and support mathematical thinking in young children in a way that is meaningful for them and that motivates them. In this article, we will advocate the value of schematising activities in early childhood education and try to clarify how educators can organise schematising activities in early childhood education. We will address these questions by means of a literature review and by describing some practical examples taken from earlier research. Before addressing these questions, a few words about the emergent approach in Dutch education.

Developmental Education

Developmental Education is the name for the Vygotsky-based approach to education in the Netherlands (see for example Van Oers, 1997; 1999; 2003; Dijk, 2003). The most important characteristics of this educational approach are:

1. Developmental education is first of all aimed at promoting children's identity development.
2. Developmental education is aimed at expanding and deepening children's current abilities. Children's development is not based on imposing new qualities in a forced manner, but on growth of their current abilities. In this process the zone of proximal development plays an important role;
3. Developmental education assigns an important role to the so-called leading activities. A leading activity is a specific type of interaction between the child and the environment that is most beneficial for developmental accomplishments. Playing is the leading activity for children aged 4-7 and productive learning is the leading activity from 7 years of age. From this developmental educational perspective, the development of children aged 4-7 must be seen as a coherent whole. In this period the transition from play to productive learning occurs. Therefore children have to develop the motive for learning during their play activities to be able to participate successfully in later learning activities. In addition, they need to develop their social competencies to be able to engage successfully in a group.
4. Developmental education is aimed at organising social cultural activities (and contents) that have meaning and personal significance for the children. Teachers have to be sensitive for children's personal interests, but at the same time they have the responsibility of enhancing the educational value of these activities. Therefore it is necessary that the teachers participate in the children's activities.

Based on this approach, a curriculum strategy has been designed in the Netherlands for the early education of children aged 4-7, known as Basic Development (Janssen-Vos, 1997). In Basic Development, mathematics at the elementary level is not just learning basic facts and skills, like operations with quantities and numbers and 'doing sums'. Teachers have to acknowledge that mathematics is basically a problem-solving activity in which the use of relations and representations of the world in the form of schematisations and models is an important feature. In the Netherlands, researchers at the Dutch Institute for the Development of Mathematics Education have developed an approach to mathematics, called Realistic Mathematics Education, which fits well into the socio-cultural approach. Realistic Mathematics Education is mainly aimed at promoting learning with understanding and promoting children's problem-solving skills with the help of

schemes and models (Freudenthal, 1973; Gravemeijer & Terwel, 2000; Van Dijk, 2002, 2003a, 2003b).

In the United States, we see a growing interest in the so-called 'modelling approach' (see for example Lehrer & Schauble, 2000; Lehrer & Schauble, 2001). Lehrer and colleagues argued that in order to understand and use models in mathematics and science, children in the early grades have to explore and familiarise themselves with different kinds of representations, schematisations or 'inscriptions' (Latour, 1990). To support the early development of modelling practices of young children, early mathematics education must therefore include aspects of geometry and not only arithmetical operations (see also Lehrer & Chazan, 1998). The research carried out by Lehrer and colleagues emphasises the importance of invention and revision of children's own representations to solve problems and demonstrates the co-development of children's mathematical thinking and their representations. Children first make representations that show a clear perceptible correspondence with the represented situation or object. Later, their representations become more symbolic and show more conventional aspects of symbolic systems (compare what Gravemeijer (1997) called the progressive schematisation process from 'model of' to 'model for'). The underlying assumption is that when children are given the opportunity to invent and revise their own representations of problem situations, they will be better able to give meaning to the problems in mathematics education (see for example Van Dijk, 2002, 2003a, 2003b) and eventually appropriate the required mathematical operations. The activity of inventing and revising representations itself can be understood as creating meaning.

Creating meaning is important to learning mathematics with understanding. The mental activity of constructing meaning and 'inscriptions' can be called 'semiotic activity' (Van Oers, 1994) and the following working definition may be used:

"the cognitive activity of reflecting on the relationship between sign and meaning, or more particularly, reflecting on the mutual relationship between the change of signs and the change of meanings." (Van Oers, 1999, p.274)

The use of signs, symbols or representations is always part of semiotic activity and semiotic activity is "an activity of working out meaning of signs referring to external (real world or ideal) objects" (Van Oers, 1994, p.24). Creating meaning and using signs or symbols are recognised as two features of mathematical thinking (e.g. thinking in addition, subtraction, multiplication and division). Pimm (1995, p.167) described mathematical thinking "as ways of thinking developed to work on mathematical forms and entities." We assume that here he was stressing the importance of making representations and interpretations of the mathematical forms and ideas in order to mediate perception of these forms. Children are better able to use symbols and representations when they understand where they stand for and when the forms used have meaning to them. Directly instructed, often meaningless, forms have no connection with children's perceptions of the objects represented. It would seem to be a good starting point in early mathematics education if we start with children's own invented representations of mathematical forms and ideas. Mathematical thinking thus involves both the use of symbols and representations and creating meaning. It requires reflecting on the interrelationships between signs (symbols) and meanings. Therefore, mathematical thinking can be conceived of as a form of semiotic activity. Early semiotic activity can, in our view, be found in schematising activities in early childhood. By schematising activity we mean every cognitive activity aimed at constructing or improving symbolic representations of a part of the physical or socio-cultural reality (Van Oers, 1994). This includes schemes, models, diagrams, graphs and also drawings, dramatising, stories and other uncomplicated symbolisations. The relevance of this semiotic activity for mathematical thinking development has recently been stated by other researchers as well (see for example Worthington & Carruthers, 2003)

Several researchers in other parts of the world have questioned children's understanding of the formal symbols taught in traditional early mathematics education. For example, Gifford (1997) reported that young children can get used to 'doing sums' by applying a memorised procedure, but that they,

“see plus and minus signs merely as prompts to count bricks or fingers, without understanding of them as symbols representing the operations of addition and subtraction.” (p.79)

Hughes (1986, p.74) wrote that the children involved in one of his studies did not use the conventional symbols to represent addition and subtraction when asked to solve problems, although they were using the conventional forms of symbols everyday on their worksheets. Pound (1999, p.26) argued that, “young children will understand and use conventional symbol systems more effectively when they know how these connect with their own ways of representing things.” Egan (1988, p.212) started at the other end when he mentioned,

“the importance of encouraging children to see the connections between the ways in which they (e.g. children) represent their own ideas and the ways in which other people choose to do.”

Semiotic or schematising activities seem, in our view, to be ways of stimulating children’s understanding of mathematics and children’s mathematical thinking. Our arguments for promoting semiotic or schematising activity in early childhood will be outlined in the next section.

Why? Some arguments in favour of schematising in early mathematics education

What are our arguments for promoting schematising activities in kindergarten? This question will be addressed in this section and answered within a socio-cultural framework.

What are our arguments for valuing schematising activities?

The Vygotsky-based approach to early education in the Netherlands offers a coherent educational system that integrates both the child’s personal constructions and the educator’s pedagogical responsibilities. Developmental Education is aimed at organising social cultural activities (and content) that have cultural meaning and personal significance for the children. Viewed from a socio-cultural stance, mathematics should be a meaningful activity embedded in meaningful contexts. It must be functionally connected with themes and with other activities in which the children take part. Mathematics has to be challenging and the connection with day-to-day reality should be preserved. Mathematical knowledge can emerge when children and teachers work together actively, as part of their communication.

Too often though, mathematics seems to be elusive for young children at the beginning of mathematics education and they show a lack of motivation or even may seem incapable of learning mathematical operations, symbolisations and quantifications in school. However, the same children often demonstrate a wide interest in counting, symbolisations and other mathematical thinking prior to entering formal schooling (see for example, Hughes, 1986; Aubrey, 1997; Munn, 1997; Pound, 1999). We believe that schematising activities in early childhood can stimulate and support the mathematical thinking in young children in a way that is meaningful for them and that motivates them. The developmental importance of schematisations as perception-based means of thinking was first emphasised by Zaporozec (1963) and later empirically substantiated by Venger (1987). Schematisations (or perceptual models) form the bridge between the concrete practical thinking of young children and the logical-symbolic thinking in later development. This bridging function as attributed to schematising is an important argument in favour of schematising in early childhood.

Another reason for attaching importance to schematising activities in kindergarten concerns the communicative function of mathematics. Mathematics is not just a matter of learning the formal and conventional ways of notating mathematical operations and standardised symbolisations, but is also concerned with communicating and with understanding the underlying meaning of the symbolised messages in the notations. Children must be made aware of this while learning math-

ematics in school. Pound (1999, p.1) suggested that meaningfulness is closely related to having real purposes for communicating when she wrote,

“if our young children are to become confident and competent users of mathematics for the twenty-first century they will need to learn to recognise mathematics as a powerful tool for communication.”

Munn (1998) also emphasised the importance of the communicative function of mathematics and of the perception children have of the communicative value of the number symbols. She argued that “there is something about the known communicative value of conventional numerals that aids children in their understanding of the abstract qualities of number” (p. 67). Further, observers in a kindergarten class often observed that young children are only willing to be involved in schematising activities when there is a communicative reason for them to do so (see Van Oers, 1994). For example, in one of our classroom visits we observed a young child who made a little village (with houses and roads) in the sandbox in school. The researchers asked the child to make a drawing of this work in order to show it to his classmates. The boy refused and did not see any reason for making and showing the drawing. Then, during the conversation with the boy, the child spontaneously started making the drawing, explaining he could show the drawing to his younger brother in a lower grade so he would already know what he is expected to do next year. In another class, we observed a child building a Christmas crib with wooden blocks. When she finished the construction, we asked the child to make a drawing of it. The child refused. Then, we asked the child again but this time we suggested making the drawing to put it later on in the box with all the other Christmas decorations so that the children who will attend this class next year, when this child will be in the next grade, could rebuild the same crib in the construction corner. Because of the difficulty of the task, the teacher gave some assistance. We can say that these children eventually make the representations, because the communicative purpose for doing so has become meaningful for them. So one important reason for stimulating schematising activities is that it provides children with means for communicating. Effective communication is an essential element in the development of mathematical thinking (see Pimm, 1987).

The third argument in favour of stimulating schematising activities in early childhood education is the enrichment it provides for the play activities of young children. In our view, play activities provide important and rich contexts for learning in young children. This will be elaborated in the next section.

When? Finding appropriate moments

If it is considered important that children get engaged in schematising activities, then the next logical question regards the moments when it is appropriate to get children involved in such activities. We will deal with this question in two steps:

- In what *context* could young children be involved in meaningful schematising activities?
- At what *age* could educators begin using schematising activities to promote mathematical thinking in young children?

In what context could young children be involved in meaningful schematising activities?

Basic Education, a curriculum strategy designed for the early education of children aged 4-7, begins with the idea that play activities are the activities that are most beneficial for developmental accomplishments for young children.

Vygotsky (1978) himself wrote about the power of play in the learning of young children. In the context of play, children are eager to learn in a way that is meaningful for them. Or, in the words of Vygotsky, play *mediates* meaningful learning of young children. Children learn to under-

stand the meanings of the world as they play with their representations of the world. Within play, children often imitate adults. "They do not just copy their actions, but they reconstruct the socio-cultural activities in their own way and make their own versions." (Van Oers, 1999b). Through these kinds of imitations, play creates a zone of proximal development within which learning takes place. "Children's greatest achievements are probably in play." (Vygotsky, 1978, p.100) "and the teacher should use the diverse contexts of play as opportunities for teaching without disturbing the play itself." (Van Oers, 1999a). Therefore, learning mathematics at the elementary level and especially the promotion of the development of mathematical thinking in young children should be stimulated within play activities.

Many other researchers have also supported the importance of play as a context for learning mathematics. For example, Aubrey (1997) mentioned the mismatch between British children's own informal invented number knowledge and the formal requirements of the school curriculum at the beginning of mathematics education. Therefore, Anghileri (2000) argued that in order to,

"transfer their (i.e. students) informal knowledge of numbers to practical applications of this knowledge in the classroom, activities can be devised to reflect situations outside school where numbers have been seen and used." (p.128).

She referred to the role-play areas in the school. Pound (1999, p.34) argued that play has several important functions for learning mathematics and promoting mathematical thinking in young children. For example, while children are playing with blocks, they explore the relationships between different lengths of the blocks and thus explore relationships that may become mathematical in due time. Likewise, they explore volume and mass while playing with water and sand. In addition, she stressed that play provides "rich learning contexts where children can reflect on previous experience and consolidate their current understanding." (p.34).

At what age could educators begin using schematising activities to promote mathematical thinking in young children?

Studies of language development have already demonstrated that the early stages of language development already serve to schematise the wealth of everyday experiences (see, for example, Tomasello, 1999). The use of even primitive categorisations permits children to structure the world in a more or less schematic way by grouping things together or creating perspectives. Schematising and semiotic activities seem to have very early precursors in children's development.

Previous studies have demonstrated that children as young as 5 years of age can be involved in simple forms of semiotic (schematising) activity within the context of their play without interrupting the play itself (see for example, van Oers, 1994; 1996; 1997; 1999a; Venger, 1986). A small scale study (Van Oers, 1994), which investigated how young (4-6 year-olds) children deal with schematising activities within their play, showed that this semiotic activity is indeed accessible for young children in their play. An important condition for the emergence of semiotic activity is the presence of a meaningful problem in the play of the children. In this study, children constructed a railway track and its surroundings (with houses, bridges etc.) and played with it for some time. The teacher participated in their play and encouraged the children to make a drawing of the track for the children of another school so that these children could rebuild the same track. While drawing, their attention was drawn to the relationship between the drawing and the track through the assistance given by the teacher. The observations showed that the play context provides starting points for early semiotic activity. The semiotic activity studied with these young children can be characterised as simple and involving static situations in which the child had to represent a perceptually accessible situation (see also Van Oers & Wardekker, 1999). It would be even more challenging to engage children in situations in which they have to make schematic representations of, for example, a melody or a story. Venger (1986) showed that children as young as 6 are able to make these kinds of schematic representations.

The question remains how educators can organise schematising activities in young children's play in order to stimulate their mathematical thinking and schematising abilities.

How? An example

How can educators stimulate the schematising activities of young children in kindergarten? In Dutch schools committed to the educational concept of Developmental Education, we encountered many interesting examples of children's schematising in the context of their play activities. Below, we will present one of these examples (taken from these schools) showing 'good practice' in early childhood education. This example is chosen because it shows, in our view, many aspects of a meaningful schematising activity and appropriate assistance of the teacher.

How can educators stimulate the schematising activities of young children in kindergarten?

The example demonstrates how the teacher can stimulate and promote schematising activity in the play activity of young children. The example is taken from our own classroom observations.

In an early years classroom where the educational practices are organised according to the principles of Developmental Education, the class is working on a unit about the Royal Family. Four children (4-6 year-olds) have built a palace [Paleis Toermalijn] with wooden blocks as a follow-up of the royal wedding of the Dutch Prince, heir to the throne, which took place a couple of weeks earlier.

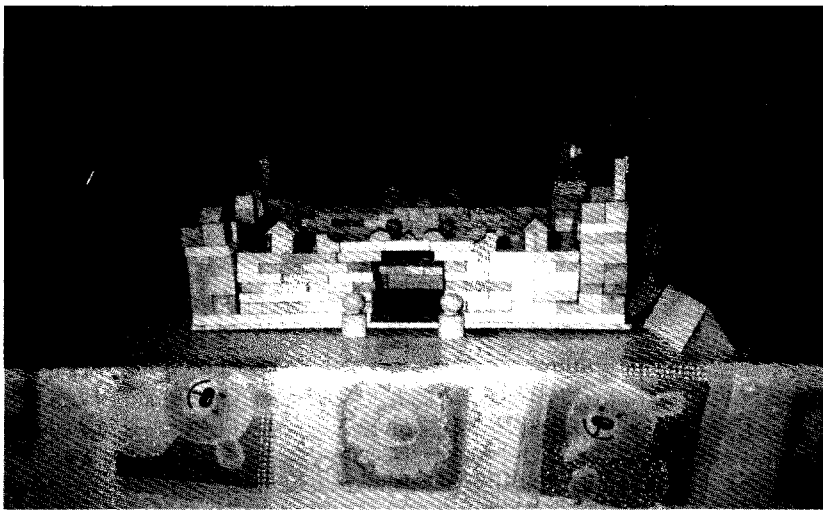


FIGURE 1

During a conversation concluding this unit, one of the children, Bas, tells that he wants to be an architect when he is grown up. It is clear that he wants to make a drawing of the palace with the assistance of the teacher. First, Bas tells the teacher he is going to draw the frontal view. He starts directly drawing the right tower at the bottom right of the paper. Then, he draws in the bricks of the tower starting at the top right. After the last brick (the exact number of bricks copied from the wooden construction), he notices that there is some space left between the bottom of the tower and the bricks on his drawing. What to do now? Bas insists on finding a solution for this problem. The teacher asks Bas if he has any suggestions. Bas proposes drawing a cross in the blank space. The teacher says that he can write the sentence "this does not belong to the tower" [dit hoort niet bij de toren]. Bas agrees and finishes the tower by drawing a flag on the top.



FIGURE 2

To draw the left tower, Bas estimates the height by comparing the height of the right tower; first with his eyes, then with his finger. Then, he copies the exact number of bricks from the construction in his drawing. Again, some space is still blank between the bottom of the left tower and the drawn bricks. Bas draws a cross in the blank space and adds an arrow for writing a text next to the cross. The teacher wants to write the text, but notices that Bas wants to write it himself. Letter by letter, he precisely copies the same text as written next to the right tower and makes no mistakes. Bas draws a new flag on the top of the left tower and this tower is ready.

Bas continues drawing a horizontal line between the two towers. He points with his finger to the wooden palace and says “that is this wall” [dat is deze muur]. Bas asks himself the question “What now? The upper part?”. “No”, he decides, “first the gate, in the middle”. Bas estimates the middle between the two towers and draws the gate. He asks the teacher to write down the word *gate* [poort] “because otherwise you don’t know” [anders weet je het niet]. The teacher writes down *gate* and Bas draws the upper part of the gate and the two merlons with triangle shapes. Then, the brick wall. Bas doubts and says “that’s much work and it’s just brickwork” [dat is veel werk en het is toch gewoon gemetseld]. He makes some gestures with his hand to indicate how you can roughly draw the brick wall, by sketching horizontal and vertical lines crossed. Bas asks the teacher to do that for him. After finishing the wall, the teacher suggests writing down the words *brick wall* [gemetselde muur]. Bas answers “yes, otherwise it looks like rope” [ja, anders lijkt het wel touw]. Then Bas draws the two soldiers and the teacher must write *the soldier* [de soldaat]. Bas: “it could also look like a wastebasket” [het zou ook een prullenbak kunnen zijn]. Finally, Bas copies the words *Paleis Toermalijn*, puts his name at the upper right side and asks the teacher to write down *the front view* [de voorkant]. (See Figure 3)

This example confirms the importance we attribute to play as a context for learning and the importance of appropriate participation of the teacher. Meaningful schematising activities seem to be accessible in early childhood play when there are functional and meaningful tools for this activity (the drawings and also writing). This example confirms the active role the teacher needs to play in these kinds of schematising activities to make them accessible and meaningful. While



FIGURE 3

working with the child and keeping track of his active involvement, the teacher has to help finding solutions for emerging problems, and the teacher has to organise and structure the schematising activity while it unfolds. We can see the way the child reflects on the interrelationship between the symbol (e.g. the drawing) and the represented object (e.g. the wooden construction) and vice versa. This reflection takes clearly place when Bas is copying the bricks (same number of bricks and same position) and notices that in his drawing some space is left over while this is not the case in the wooden construction.

Bas is not just drawing, he is schematising; he is constructing and improving the symbolic representation of the construction. First, Bas' representation shows a perceptible correspondence with the represented object. For example, he exactly draws the bricks as they look in reality; he exactly copies the number of bricks in the wall of the two towers. Then, when drawing the brick wall, Bas decides not to draw all the bricks, but symbolises the wall in a way that is meaningful for him and is less time consuming. The teacher suggests to write the words *brick wall* to communicate the symbolised meaning of his drawing. Further, Bas shows an understanding of the communicative function of schematisations. Several times he asks the teacher to write the words to communicate the meaning of the drawn elements; for example, after drawing the gate: "can you write the words, otherwise you don't know"; after drawing the brick wall: "otherwise, it looks like rope"; after drawing the two soldiers: "it could also look like a wastebasket". It is the teacher who can give him the assistance he needs to complete the communicative function of his drawing.

In our view, it seems important that teachers become receptive to moments where schematising emerges in the play of the young children. Then, teachers can pay attention to the schematising activity and stimulate the children involved to complete this activity and to emphasise the importance of the communicative function of the scheme. By doing so, repetitively, children will probably invent new symbols to include in their scheme or drawing, symbols with a more general meaning.

The example above demonstrates how a teacher can promote schematising in play and stimulate semiotic activity (including the reflection on the relation between the sign and meaning). We encountered many such examples in our classroom observations and it seems reasonable to assume that this is a promising approach. However, more research is needed in order to corroborate our ideas.

Conclusions and a glance at the future

The research regarding the development of thinking provided a number of good reasons for giving schematising activities a more prominent place in early childhood education, especially for the promotion of mathematical thinking. Two important findings can be reported from the study conducted by our own research group and elsewhere with respect to the introduction of schematising in early childhood:

1. precursors of semiotic activity can be found from a very early moment in (language) development; schematising (as an explicit form of semiotic activity or mathematising) is accessible for young children (from the age of 5) when it has a clear communicative function;
2. in order to be meaningful, the schematising activity must be embedded in activities that are developmentally appropriate; for young children this means that schematising should be embedded in the context of play; studies in our own research group have demonstrated that it is possible to integrate schematising activities in play without impairing the quality of play itself.

Over the past decades many schools in the Netherlands committed to the Developmental approach, have implemented this view on schematising and demonstrated that it can be successfully applied in practice, as shown by the examples given.

However, the research in our own research group has thus far focused mainly on case studies that are intended to clarify the main characteristics of this semiotic activity in a variety of contexts, as well as articulate some of the conditions for its realisation. In this research, mainly static forms of schematisation have been used (i.e. schematisations of static situations, like buildings and maps). We think it is now appropriate to expand our scope further and include dynamic and three-dimensional situations as well. Moreover, it is important to test the presumed potentials of schematising for mathematical thinking in a more robust experimental design. We are dealing with this question in a current research project. Results will be presented in the future.

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